



Instituto Superior Técnico

**Elaborated in the context of the BLUE project**

## **Recruitment Task nº 1**

Engine component research

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# 1 Introduction

This document will focus on the different variants and design requirements of the turbomachinery found in a typical jet engine as well as the combustion chamber and the nozzle. In the previous project a basic overview of the entire turbojet engine has already been presented, this allows us to now dive straight into some more complex topics.

## 2 Compressor

The compressor's objective is to increase the pressure of the flow, it does this by inputting work into the flow in "a rotational manner" (synergy with the turbine and shaft implementation). Work here is understood as the increase of kinetic energy of the flow which can then be converted into potential energy via a diffuser. The reason why this "rotational manner" is mentioned is to intuitively understand that the added kinetic energy will come under the form of a new tangential velocity component which we call swirl, and it's this very swirl which will then be used to attain the aforementioned pressure increase.

### 2.1 Axial compressor

Having established the base design requirement of the compressor we may now start addressing the solutions with the axial compressor. This implementation is made up of stages each with a rotor and stator sections. Both of these are made up of airfoil shaped blades with the first one being connected to the rotating concentric shaft and the second one mounted to the engine case (static hence "stator").

The blades of the rotor stage are setup in a way that the relative velocity of the air (airflow velocity before the compressor together with the rotation of the blade) aligns itself with the chord of the airfoil which then applies a force on the flow thus accelerating it tangentially (and increasing its kinetic energy which in turbomachinery  $\equiv$  increasing  $P_{\text{stagnation}}$ ). The stator is also made up of blades but these are static and decelerate the flow tangentially but without removing the energy from the flow (if we disregard losses). This creates a static pressure increase which is the objective of the entire process.

Below we find this concept explained with the velocity triangle :

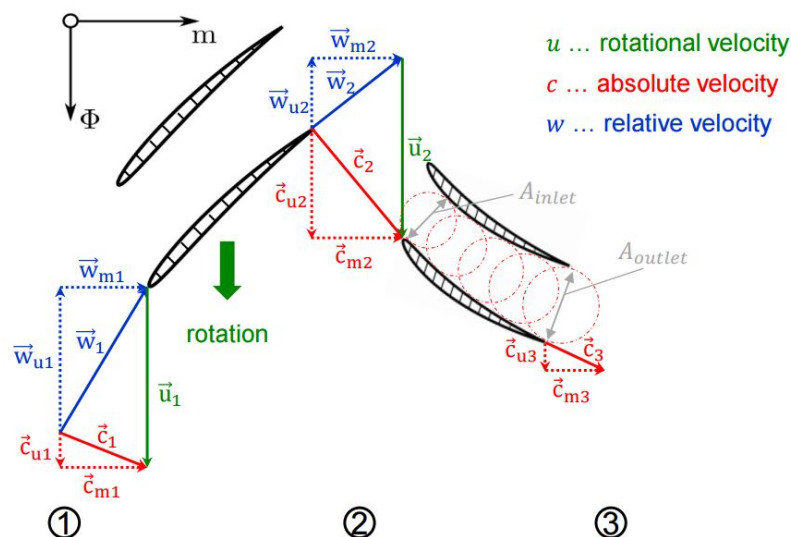


Figure 1: Axial compressor rotor velocity vector [4]

Note the increasing area between the blades to decelerate the flow in both the stator and the compressor, this leads to a 0 work compression via the the same logic of the diverging nozzle.

- The equation which describes this process best can be derived from the conservation of angular momentum  $D$  [1] [4]:

$$\frac{d}{dt}D_0 = \sum M_0$$

And then removing the cross product by introducing the tangential component of the absolute velocity,

$$\vec{D} = m \cdot \vec{c} \times r \Rightarrow |D| = m \cdot c_u \cdot r \Rightarrow \left| \frac{dD}{dt} \right| = \dot{m} \cdot c_u \cdot r$$

The applied momentum to the flow can then be quantified by,

$$M = \dot{m}_2 \cdot c_{u2} \cdot r_2 - \dot{m}_1 \cdot c_{u1} \cdot r_1 = \dot{m} \cdot (c_{u2} \cdot r_2 - c_{u1} \cdot r_1)$$

Using the definition of power (P), work per unit mass ( $w$ ), and rotational velocity ( $\omega$ )

$$P = M \cdot \omega = \omega \cdot \dot{m} \cdot (c_{u2} r_2 - c_{u1} r_1)$$

$$w = \frac{P}{\dot{m}} = \frac{\omega \cdot \cancel{\dot{m}} \cdot (c_{u2} \cdot r_2 - c_{u1} \cdot r_1)}{\cancel{\dot{m}}} \quad \wedge \quad \omega \cdot r = u$$

We end up with the final equation :

$$w = c_{u2} \cdot u_2 - c_{u1} \cdot u_1 \tag{1}$$

From here it is clear the act of adding energy to flow via the increase of swirl (tangential velocity) has a few design tricks which can be used :

- Optimize  $u_2$  which corresponds to increasing the rotational velocity at the rotor exit. This is done by increasing the meridional radius of the rotor blade as we reach the exit (2 station as seen in 1).
- Optimize  $c_{u2}$  (for the same  $|\vec{c}_2|$ ). An option for this is the higher turning of the flow. It can be done via an increase in camber or performance of the airfoil which of course leads to higher forces and higher energy transferred. It can also be related with the increasing area between blades as this decreases  $w_{u2}$  and  $c_{u2} = u_2 - w_{u2}$ .
- Minimize  $u_1$ . Doing the opposite of what was mentioned beforehand and starting with a very small meridional radius.
- Minimize  $c_{u1}$ . What is called "pre-swirl" is a consequence of the conditioning of the flow before it enters the compressor, ideally this would be as negative as possible but realistically it tends to be slightly positive.

While reviewing the work equation it is also obvious that the work produced by a certain blade section is dependant upon the radius in the following manner (T for tip, H for hub) :

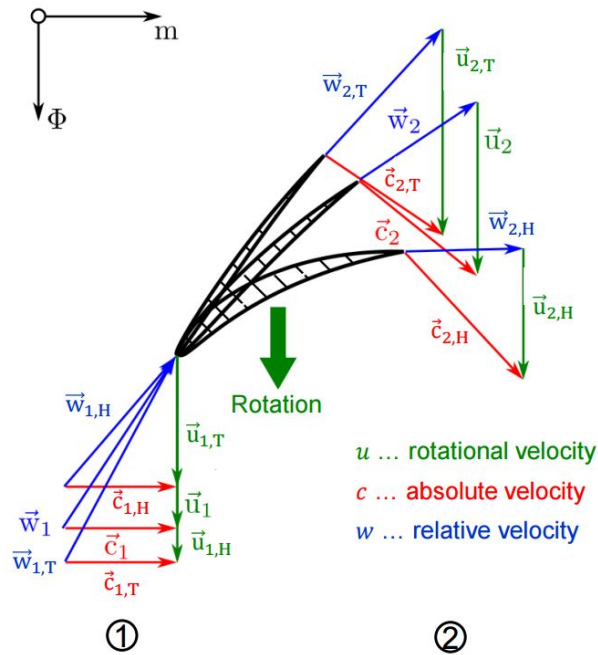


Figure 2: Radius increase effect on the velocity triangle [4]

In order to avoid unwanted internal momentums twist is then added to the blades in order to make it so that  $c_u \neq f(r)$ .

By inspecting the shape of the compressor/fan blades (fan also inputs some work into the flow) we can quickly see how some of these tricks are used, especially regarding the increasing meridional radius, camber and twist. Do note that the increase in aerodynamic performance of the airfoil can also bring it closer to stall (higher effective angle of attack) which severely limits this approach.

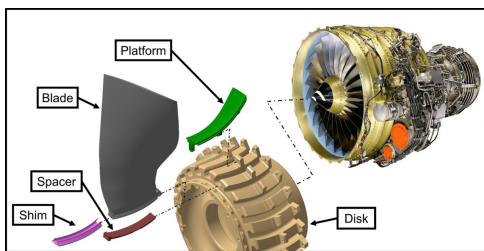


Figure 3: Fan blade

CFM56-3 737-3/4/500 High pressure compressor blades scaled 1:1 for boroscope purposes and comparison purposes only. Scale is 1:1 if printed on A4



Figure 4: Compressor rotor blade

The final parameter on which the actual work done by the compressor  $\dot{W} = w \cdot \dot{m}$  is the mass flow rate  $\dot{m}$ . Its value is one of the key design points of the engine and it will define essential factors such as the intake/inlet size and also affect the performance of the compressor as it will be seen later on in the compressor map.

## 2.2 Radial compressor

The radial compressor is similar to the axial compressor in concept, but with some key differences. Flow enters the impeller, the radial equivalent of the rotor of an axial stage. As the air passes through the rotating blades, it is turned outward, with a significant rise in meridional radius.

This radial motion subjects the fluid to centrifugal acceleration, increasing its kinetic energy (tangential component increase). The impeller conveys the flow through blade passages that allow a larger effective area change than in an axial compressor stage. Like in the axial case, the impeller is then followed by a diffuser made up of static blades predisposed on a "plate" (image further up ahead) where the swirl is converted into potential energy under the form of a static pressure increase.

The equation used for the radial compressor is very similar taking up the following form :

$$w = c_{\theta_2} \cdot u_2 - c_{\theta_1} \cdot u_1 \quad (2)$$

Diagram representation :

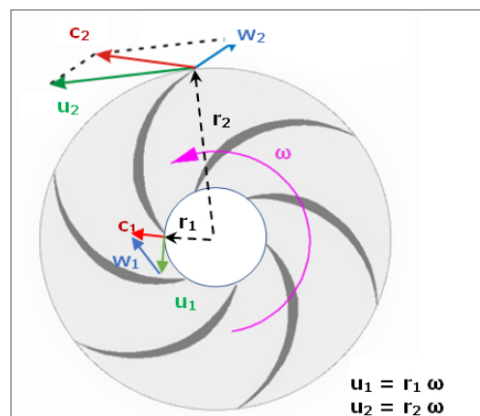


Figure 5: Velocity triangle for the impeller

Note that  $|c_{\theta}| = |\vec{c} \times \vec{r}|$  and note the difference between  $u_2$  and  $u_1$  when compared to the axial case.

From what we've discussed until now, it seems that the radial compressor is, independently of the scenario, the clear winner in a contest between both solutions. This however is not true due to two main problems with the implementation, the weight per stage being much greater and that it doesn't allow for large amounts of mass flow  $\dot{m}$  to go through which naturally greatly impacts  $\dot{W} = \dot{m} \cdot w$ . The reasoning for the mass flow limitation is that in order to accelerate outward, the mass flow is limited to the centre part of the impeller as can be seen in 5.

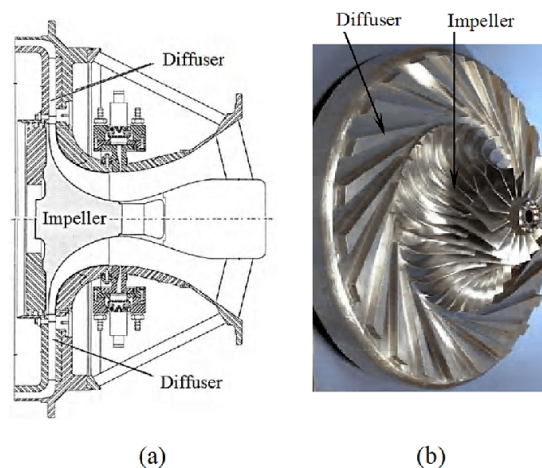


Figure 6: Centrifugal compressor stage

## 2.3 Design comparison

In order to better understand what the process is when choosing the design which fits the project in question most the best way is to visualize it with a table :

Table 1: Comparison between Axial and Centrifugal Compressors

Axial Compressor	Centrifugal Compressor
Higher mass flow rates	Higher pressure rise per stage
Smaller length per stage	More compact radial layout
Straighter path of the airflow	Better stability margin
More scalable, lower rotational speeds	Simpler blade shapes and robust design

## 2.4 Compressor map and reduced quantities

Compressors operate under widely varying inlet conditions. To compare performance consistently we enforce *Mach number similitude* (axial and circumferential Mach numbers preserved) and use reduced or corrected parameters. The final definitions are:

$$\dot{m}_{red} = \frac{\dot{m} \sqrt{T_t}}{p_t A} \quad n_{red} = \frac{nD}{\sqrt{T_t}}$$

An example of how this simplification can help considering a compression from point **E** → **A**

$$p_{tA} = f(p_{tE}, T_{tE}, n, \dot{m}_E)$$

Can also be characterized by the pressure ratio,

$$\Pi = \frac{p_{tA}}{p_{tE}} = f\left(\frac{\dot{m}_E \sqrt{T_{tE}}}{p_{tE} A_E}, \frac{N}{\sqrt{T_{tE}}}\right)$$

In a constant-geometry turbo machine the geometric area  $A$  and the diameter  $D$  are commonly omitted from the similarity treatment. The two main axes of a compressor map then become a total pressure ratio (or stage pressure ratio) and the reduced mass flow. This produces compact compressor maps where each speed line corresponds to a constant  $n_{red}$ .

In order to keep the units (simplifies the engineer's job and intuition of the map's meaning), the reduced quantities are often normalized to a reference state (index "corr", usually ISA sea level  $p_{ref} = 101.325$  kPa,  $T_{ref} = 288.15$  K) [4]:

$$\dot{m}_{corr} = \dot{m} \frac{\sqrt{T_t/T_{ref}}}{p_t/p_{ref}} \quad n_{corr} = n \sqrt{\frac{T_{ref}}{T_t}}$$

Inserting the isentropic relations into the reduced mass flow definition yields the well-known dependence on the local Mach number. Normalizing to the choked value (starred) gives the relation used to show choking behaviour:

$$\frac{\dot{m}_{red}}{\dot{m}_{red}^*} = \text{Ma} \frac{\left(1 + \frac{\gamma-1}{2} \text{Ma}^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}}{\left(\frac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}}$$

Here  $\dot{m}_{red}^*$  represents the reduced mass flow at the critical (choked) condition where  $\text{Ma} = 1$  in the throat (narrowest cross-section). When choking occurs the reduced mass flow cannot be increased further, setting a practical limit on the achievable compressor (and engine) mass flow and power. The phenomenon of choking will be expanded upon later on in the nozzle section.

### Explanation of the compressor map:

- Plotting  $\Pi_t$  vs.  $\dot{m}_{red}$  (or  $\dot{m}_{corr}$ ) with speed lines of constant  $n_{red}$  yields the typical compressor map.
- The *surge line* and the *choke line* bound the usable or stable map area. Choking corresponds to the right-hand boundary (maximum  $\dot{m}_{red}$ ) and surge to the left-hand boundary (minimum stable  $\dot{m}_{red}$ ).
- Corrected parameters remove distortion due to ambient pressure/temperature changes so the map is independent of test condition.
- Circular areas or "islands" are zones with a certain operating efficiency  $\eta$ .

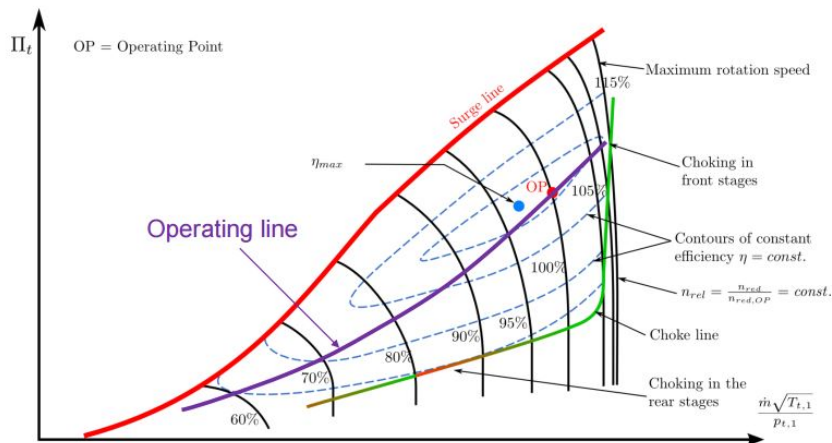


Figure 7: Compressor map

The surge line defines the boundary between the stable and unstable region of engine operation. From there on instability effects such as mass flow and pressure oscillations begin appearing. This is due to the fact that at or near this line the aerodynamic loading on the blades begins nearing its maximum value before stall. These localized stall events may start on one specific blade and then propagate to the next one leading to a rotating stall.

In the case of the compressor, the flow evolves with a positive pressure gain which means that in case of stall reverse flow from the combustion chamber to the compressor may occur which a catastrophic effect.

It is noteworthy that stall may not always lead to a surge but no surge can occur without first there being a stall.

In order to quantify how close we are to this scenario the surge margin ( $SM$ ) was created :

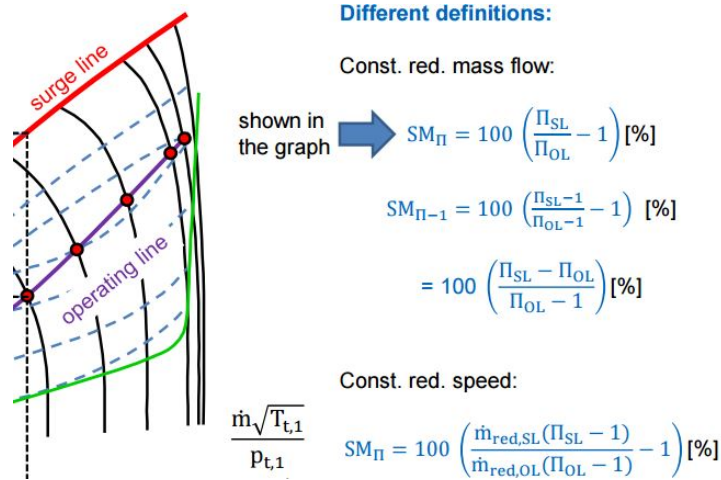


Figure 8: Surge Margin definitions

Choking will be better explained further down the line the nozzle section but for now we may simply define it as the point where the constant speed line becomes vertical which is the same as saying that from that point on, for the same speed line, the reduced mass flow remains constant.

### 3 Turbine

The turbine is almost an exact mirror of the compressor, it has rotors and stators but with the key difference of the rotor now extracting swirl from the flow and the stators converting the potential energy of the flow into kinetic energy (create a tangential component). So we just take the same equations 1 and 2 to describe the work of an axial and centrifugal turbine respectively.

A new important addition to the equation repertoire is the power relationship between the turbine and compressor since they're connected via a mechanical shaft :

$$P_{comp} = \eta_{mech} \cdot P_{turbine} \quad (3)$$

The velocity diagram for the turbine elucidates how the opposite is wanted from the compressor, we now want  $w$  to be as negative as possible.

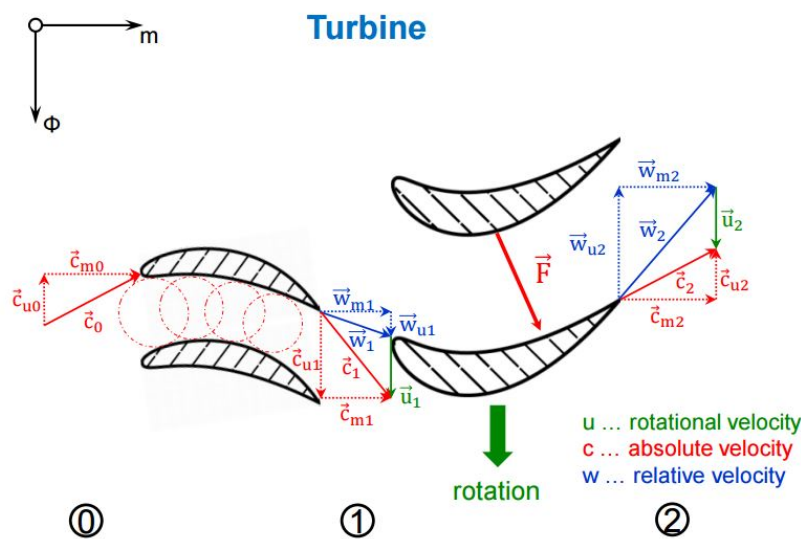


Figure 9: Turbine velocity vector

Note that the stator now comes before the rotor in order to introduce swirl to the flow which ideally

comes out with minimal rotation from the combustion chamber to avoid certain problems. Also, the area between blades decreases to accelerate the flow as much as possible.

### 3.1 Axial vs Radial

In the same manner of the previous section, a table is now presented with the main defining characteristics of both:

Table 2: Comparison between Axial and Centrifugal Turbines

Axial Turbine	Centrifugal Turbine
Better efficiency at higher flow rates	Higher specific work per stage
Longer, more aerodynamically complex design	Compact radial configuration
Directional flow aligned with shaft axis	Flow turns outward through radial blades
Scales well for multi-stage expansion	Fewer stages, simpler construction
Often used in large gas turbines	Common in smaller turbochargers and APUs

### 3.2 Turbine map

The turbine map could be represented in the same manner ( $\Pi_t$  vs  $\dot{m}_{red}$ ) but this would result in a densely packed set of curves. For a better readability of the turbine performance map, a better alternative for the parameters is the reduced turbine power and the product of reduced mass flow and reduced speed.

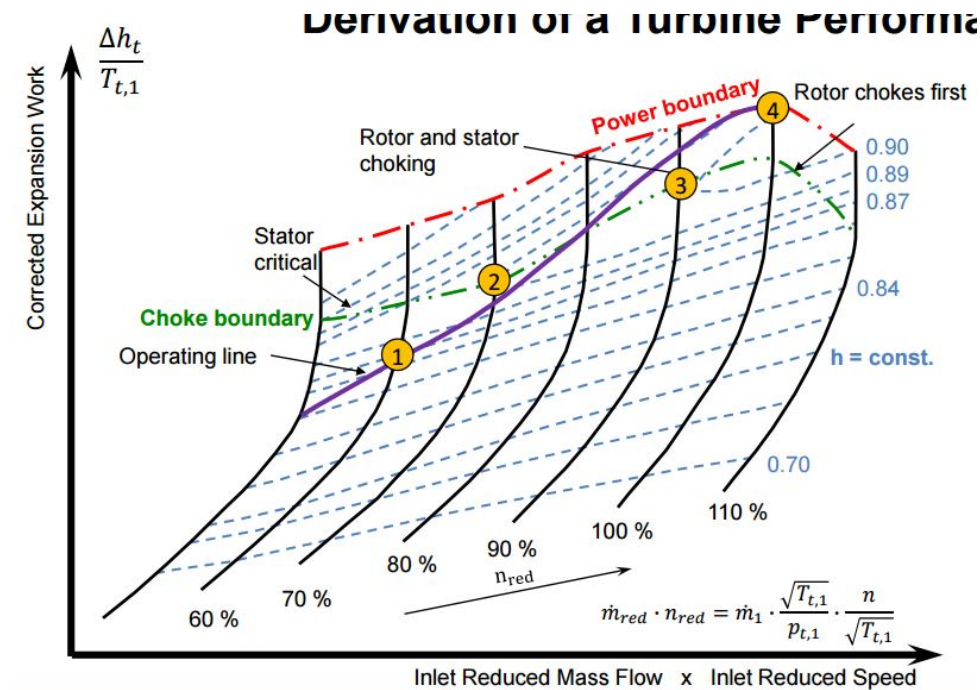


Figure 10: Turbine map

Due to the nature of the turbine being an extractor of power, stall is no longer a concern, instead what we worry about is extracting power. As it was previously stated, the area between blades decreases in the turbine which accelerates the flow but it may accelerate it up the point where choking occurs (refer to the convergent divergent nozzle section for the theoretical basis).

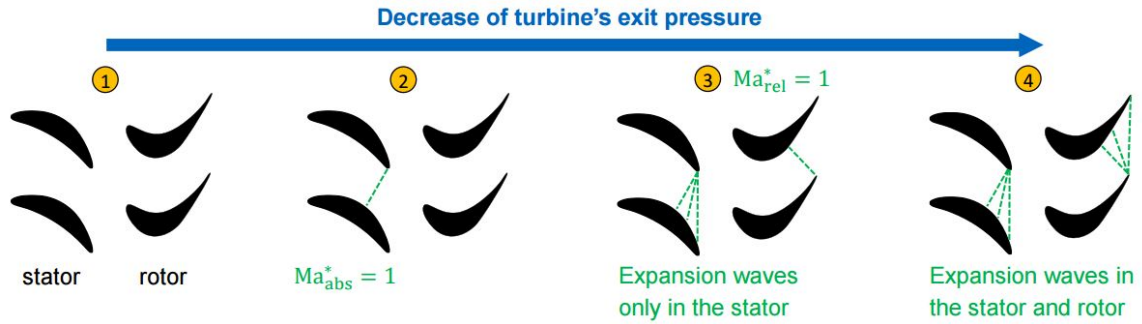


Figure 11: Turbine choking referring to fig. 10

- In case 2, the acceleration due to the area reduction leads to sonic flow at the in-between stator blade throat which limits the maximum mass flow rate hence being called the choke boundary.
- In case 3, expansion waves caused by an area increase of the now sonic flow leads to acceleration into supersonic values which further increases the value of  $c_1$  before entering the rotor thus allowing more work to be extracted
- In case 4 expansion waves appear at the rotor exit, this may seem something which would decrease the power extraction as it would increase  $c_2$  but remember that this acceleration occurs in the relative plane of motion,  $\vec{w}_2$ . Since  $c_2 = u_2 - w_2$  in absolute terms, this increase implies a smaller  $c_2 \cdot u_2$  and hence more work extracted.

The main idea to be taken away is that there is a certain amount of maximum power that can be extracted ( $P = \dot{W}$ ) and this happens due to choking of the flow. Another main characteristic of the turbine is its resistance to heat but this will be dealt with in the thermodynamic cycle section.

## 4 Combustion chamber

A short summary of the combustion chamber would be that its objective is to stoichiometrically mix the compressed flow (from the compressor) with fuel and ignite it. A useful albeit basic equation to describe the energetic gain in the combustion chamber is :

$$\dot{m}_3 \cdot c_{p,3} \cdot T_{t,3} + \dot{m}_f \cdot (\eta_{cc} \cdot H_l + h_{t,f}) = \dot{m}_4 \cdot c_{p,4} \cdot T_{t,4} \quad (4)$$

With  $\eta_{cc}$  being the combustion chamber efficiency (determined by the design),  $H_l$  is the fuel combustion enthalpy ( $\frac{kJ}{kg}$ ),  $h_{t,f}$  the fuel enthalpy (contribution from its temperature, typically can be considered 0) and  $\dot{m}_f$  the mass flow rate. The remaining quantities were already discussed in the previous project but a small revision will be made in the thermodynamic cycle section.

### 4.1 Design requirements

As it was previously stated  $\eta_{cc}$  depends on the design of the combustion chamber. Here a few design requirements :

- Conversion of the chemical energy of the fuel into thermal energy with a high combustion efficiency
- Stable combustion under: High mass flow rates / short residence time in the combustion chamber at all operating conditions (pressure and mass flow variations) → Large working range: 0,3 to 45 bar, -30 to 1750 °C
- Good ignition, re-ignition, and acceleration behaviour
- Provision of an acceptable temperature level and distribution at the turbine inlet
- Low pressure loss
- Short overall length, low weight

- Structural integrity under high pressures (thrust transmission)
- High service life: 20,000–30,000 hours (civil), 2,000 hours (military)
- Minimal emissions: Unburned hydrocarbons (UHC), Carbon monoxide (CO), Nitrogen oxides (NO and NO<sub>2</sub>), Soot (impact on atmospheric transparency)
- Compatibility with various fuels and additives: Jet A-1, JP-4, JP-5, Jet B, AVGAS, synthetic fuels (“designer fuels”); anti-icing, antibacterial, and anticorrosion additives

## 4.2 Design types

### 1. Can type combustor

A self-contained cylindrical chamber (or several of them in a ring) each with its own fuel-injector, igniter and liner. Primary air enters the can, mixes with fuel and ignites. Secondary air then enters around the liner to cool and dilute. Advantages: simple to design and test (you can knock out one can at a time), easy maintenance (remove a single can). Disadvantages: heavier, larger frontal area, higher pressure drop ( $\approx 7\%$ ). [3]

### 2. Cannular (or “can-annular”) combustor

A hybrid between “can” and “annular” types: you still have discrete liners (“cans”) each with their fuel injectors, but all share a common outer annular casing. The combustion zones are separate, but they can communicate via holes or tubes so the flow and temperature distribution becomes more uniform. Lighter and lower pressure drop ( $\approx 6\%$ ) than pure “can” type, but a bit more complex to maintain. [3]

### 3. Annular combustor

A continuous ring (annulus) rather than discrete cans. There’s one continuous liner and casing around the circumference. Benefits include the lowest pressure drop ( $\approx 5\%$ ), shorter and lighter design, more uniform exit temperature profile (excellent for downstream turbine stages). The downside: requires full-scale testing and can be more demanding to design. Most modern aero-engines use this type. [3]

### 4. Double Annular (and Multi-Annular) combustor

This is a variation of the annular type with two (or more) concentric combustion zones around the ring: a “pilot” zone that runs during low power, and a “main” zone that comes in during high power. Helps with emissions control ( $NO_x/CO_2$ ) and flexibility across operating regimes. [3]

Here is the visualization of the different types :

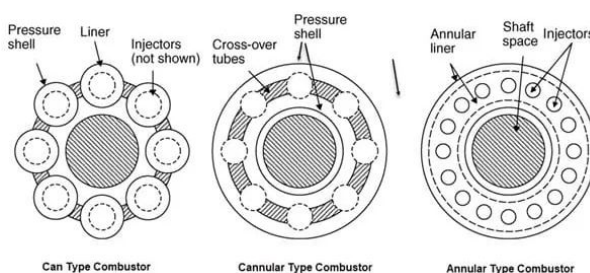


Figure 12: Types of combustion chamber design

## 5 Nozzle

The nozzle is the last component of the engine. It manipulates the velocity of the flow by converting kinetic energy to its potential form (in terms of static pressure) or vice-versa.

Let us, before going deeper into the subject, start with some theory, namely with the mass flow conservation :

$$\dot{m} = \text{const.} = \rho A c \quad (5)$$

And so in the incompressible regime where  $\rho \approx \text{const.}$ , this implies that  $A \propto \frac{1}{c}$  and so a converging nozzle will lead to an acceleration of the flow and vice-versa.

Energetically speaking, this increase in kinetic energy does not come from nowhere and it is taken away from the potential energy of the flow via the conservation of total energy of the flow (1<sup>st</sup> law of thermodynamics):

$$\sum \dot{Q} + \sum \dot{W} + \sum \dot{m} \left( u + p \cdot v + \frac{c^2}{2} + g \cdot z \right) = \frac{d}{dt} \left[ \sum m \cdot (u + e_{\text{kin}} + e_{\text{pot}}) \right]$$

$$\frac{d}{dt} = 0$$

$$\dot{Q} + \dot{W} = \left[ \dot{m} \left( h + \frac{c^2}{2} + g \cdot z \right) \right]_{\text{out}} - \left[ \dot{m} \left( h + \frac{c^2}{2} + g \cdot z \right) \right]_{\text{in}}$$

$$q + w = h_{\text{out}} + \frac{c_{\text{out}}^2}{2} + g \cdot z_{\text{out}} - h_{\text{in}} - \frac{c_{\text{in}}^2}{2} - g \cdot z_{\text{in}}$$

$$q = \frac{\dot{Q}}{\dot{m}}, \quad w = \frac{\dot{W}}{\dot{m}}$$

And using the definition of total enthalpy:

$$h_t = h + \frac{c^2}{2} + g \cdot z$$

Which leads to the final expressions that describe one dimensional isentropic flow of a perfect gas [2]:

### Isentropic Flow Relations

$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2, \quad \frac{p_0}{p} = \left( 1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{\gamma}{\gamma-1}}, \quad \frac{\rho_0}{\rho} = \left( 1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{1}{\gamma-1}}$$

$$M = 1 \Rightarrow \frac{T^*}{T_0} = \frac{2}{\gamma+1}, \quad \frac{p^*}{p_0} = \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}, \quad \frac{\rho^*}{\rho_0} = \left( \frac{2}{\gamma+1} \right)^{\frac{1}{\gamma-1}}$$

### Mass Flow per Unit Area

$$\frac{w}{A} = \sqrt{\frac{\gamma}{R}} \frac{p_0}{\sqrt{T_0}} \frac{M}{\left( 1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$\left( \frac{w}{A} \right)_{\text{max}} = \frac{w}{A^*} = \sqrt{\frac{\gamma}{R}} \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \frac{p_0}{\sqrt{T_0}}$$

### Area Ratio Relation

$$\frac{w/A}{(w/A)_{\text{max}}} = \frac{A^*}{A} = \frac{1}{M} \left[ \frac{2}{\gamma+1} \left( 1 + \frac{\gamma-1}{2} M^2 \right) \right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$M = \frac{A^*}{A} \left[ \frac{2}{\gamma+1} \left( 1 + \frac{\gamma-1}{2} M^2 \right) \right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

or, rearranged,

$$M = \sqrt{\frac{2}{\gamma-1} \left[ \left( \frac{A}{A^*} \right)^{\frac{2(\gamma-1)}{\gamma+1}} \left( \frac{\gamma+1}{2} \right) - 1 \right]}$$

Note that  $A^*$  is the nozzle area for which choking conditions occur (or in other words the maximum mass flow rate through the nozzle is achieved with  $M = 1$  at the throat).  $w/A$  corresponds to the volumetric flow rate divided by the area which is equivalent to  $\dot{m}$ .

The key intuition to take away from all of this information is that for subsonic flow, a cross sectional area decrease will lead to the acceleration of the flow (and vice-versa) and that for supersonic flow, a cross sectional area increase will lead the acceleration of the flow (and vice-versa).

## 5.1 Application and types

Now that we've established how the nozzle cross-sectional area affects the flow, we can discuss the main nozzle types utilized in propulsion applications.

The convergent nozzle sees a reduction in area throughout its length. Usually, this is done to accelerate the flow (hence working in the subsonic regime) in order to optimize the exit velocity  $c_E$  and hence the output thrust ( $\dot{m}_E \cdot c_E$ ), this deduction was done in the previous project).

The convergent-divergent nozzle allows further acceleration by taking the flow up to sonic conditions at the throat and then, once the flow is now in the supersonic regime, the diverging area after the throat leads to supersonic acceleration and so, even if the mass flow has already reached a maximum ( $\dot{m} = \text{max}$ ).

This type of nozzle also has a special behaviour at the exit depending upon the ambient pressure unto which it expels its flow. Also shockwaves may develop at or after the throat (in case a sonic throat is reached). An important intuition for nozzles is that the flow of information does not only go downstream. In case the ambient exit pressure decreases, the pressure gradient will increase and a subsonic throat will become supersonic.

Effectively, each different scenario (subsonic throat, sonic throat, shockwaves at the throat, shockwaves at the diverging nozzle, underexpanded flow and overexpanded flow) can be described and "predicted" via the conditions of the reservoir, the ambient exit pressure, the gas parameters (such as  $\gamma$ ) and the throat and exit areas. This can be seen in the following graph :

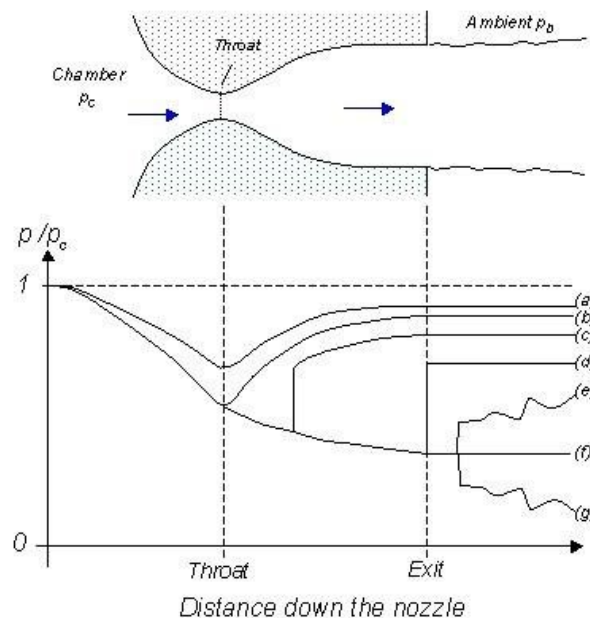


Figure 13: Pressure ratio graph for a CD nozzle

## 6 Thermodynamic cycle

In order to finalize this report, I wanted to go over some very important thermodynamic cycle parameters. The basic deductions have already been made in the previous report so now we can start straight from the graph. Here is a representation of the non ideal cycle :

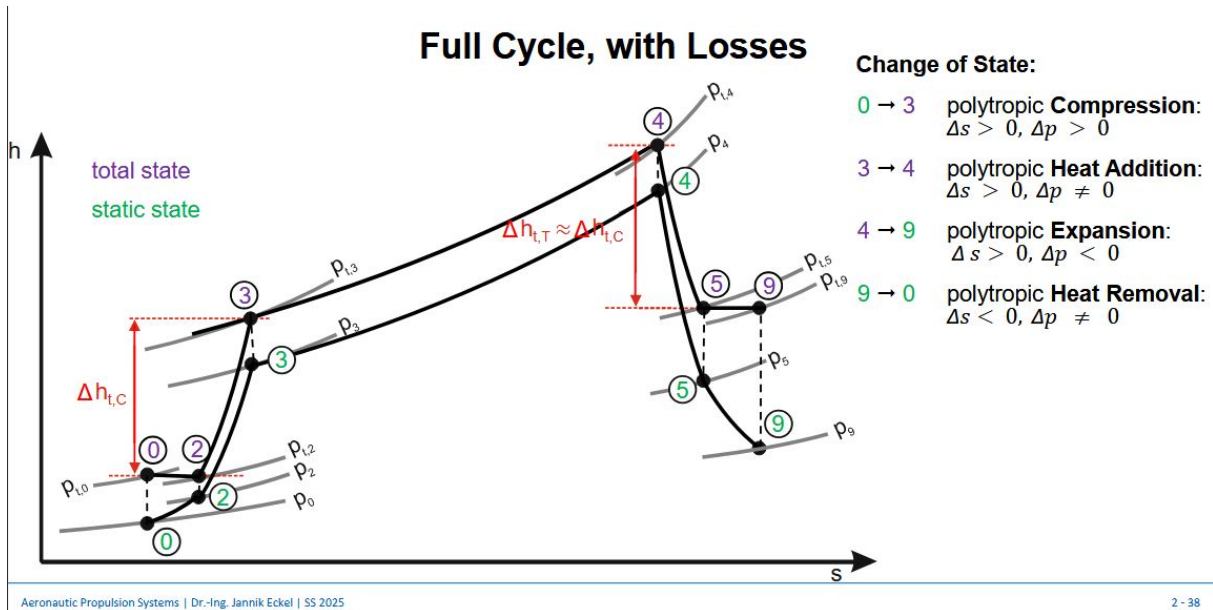


Figure 14: Real Joule-Brayton cycle

Firstly, from the previous report, the thermal efficiency of the cycle comes from two important temperatures whose ratio can also be given as a pressure ratio ( $\Pi_t$ ) via isentropic relations :

$$1 - \left(\frac{T_{t,9}}{T_{t,4}}\right) = 1 - \left(\frac{1}{\Pi_t}\right)^{\frac{k-1}{k}} = \eta_{th} \quad (6)$$

Here we can see that having a value of  $T_{t,4}$  as high as possible is beneficial for the thermal efficiency (note that this is a cycle efficiency and appears even for the ideal cycle as a cost of heat to work conversion). This however would present some challenges in turbine material design which is why the cycle efficiency has a ceiling which it cannot break. Another problem with increasing  $T_{t,4}$  is the emission of  $NO_x$  gases which will increase with higher burn temperatures in the combustion chamber.

Note also how the work per mass flow rate ( $\frac{W}{\dot{m}} = \Delta h_t = c_p \cdot \Delta T_t$ ) that powers the compressor ( $\Delta h_{t,c}$ ) is approximately (losses, quantified by  $\eta_{mech}$ ) equal to the turbine work ( $\Delta h_{t,t}$ ) as it was previously stated.

The compression and expansion processes both have an efficiency which may come in two forms, starting with isentropic efficiency which compares the real and isentropic cases in terms of total energy needed or extracted :

$$\eta_{is, C} = c_p \cdot \frac{T_{t,3, is} - T_{t,2}}{T_{t,3} - T_{t,2}} \quad (7)$$

$$\eta_{is, T} = c_p \cdot \frac{T_{t,4} - T_{t,9}}{T_{t,4, is} - T_{t,9}} \quad (8)$$

The polytropic efficiency does a more "step-by-step" approach, more suited to multi-stage compressors and adjusts the pressure-temperature relations directly :

$$\frac{p_{out}}{p_{in}} = \left(\frac{T_{out}}{T_{in}}\right)^{\frac{n}{n-1}} \quad (9)$$

With the exponent being given by :

$$\frac{n}{n-1} = \eta_{pol,C} \cdot \frac{\gamma}{\gamma-1} \quad (10)$$

$$\frac{n}{n-1} = \frac{1}{\eta_{pol,T}} \cdot \frac{\gamma}{\gamma-1} \quad (11)$$

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## 7 References

### References

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